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**Abstract**

This paper critically examines the Euro method usage for the purposes of updating supply and use tables (SUTs) and/or input-output tables. Its known restricted applicability to only unnecessarily aggregated and symmetric SUTs (not their underlying rectangular versions) is already an issue of concern. However, by studying analytically the nature of Euro's adjustments of the SUT elements and empirically assessing some of its underlying assumptions, including newly revealed ones, it is concluded that the Euro method is a largely ad-hoc updating procedure. Its recently claimed superiority over the generalized RAS approach (GRAS, or SUT-RAS) in the absence of industry output is challenged. It is shown that applying the standard GRAS with exogenously given estimates of industry outputs still outperforms the Euro method.

**Keywords:** Updating supply and use tables, non-survey techniques, Euro method, GRAS method

**JEL Classification Codes:** C02, C61, C80, D57

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# 1 Introduction

The Euro method for updating input-output tables (IOTs) was developed by Prof. Joerg Beutel (Beutel, 2002; Eurostat, 2008, Chapter 14.4.4) and extended to supply and use tables (SUTs) in Beutel (2008). The later extension is also sometimes referred to as the SUT-Euro method (United Nations, 2018; Valderas-Jaramillo et al., 2019).[1] The original version of the Euro method was used, at least in the past, by Eurostat (Eurostat, 2008), while "the SUT-EURO method has been used *extensively* by Eurostat in the estimation of European SUTs and IOTs" (United Nations, 2018, p. 485, italics added). This is surprising since the empirical evaluations carried out by Temurshoev et al. (2011) and Temurshoev and Timmer (2011) (henceforth, TWY and TT, respectively) showed that the Euro method was among the worst performing updating methods.

According to Eurostat (2008, p. 475), "the main advantages of the Euro update procedure are:

1. robust update procedure at low costs,
2. limited data requirements,
3. only official sources are used for the update,
4. integrated estimation of all four quadrants of the input-output table,
5. no arbitrary changes of input coefficients,
6. row and column totals for intermediate consumption are derived within the procedure,
7. structural composition of final demand is estimated during the iteration, and
8. consistency of supply and demand is provided by [the] input-output model."

Two other crucial statements in favor of the Euro method are the following:

9. "In contrast to the RAS procedure, this method guarantees that innovative sectors gain in relative importance in all activities, while declining sectors lose in importance everywhere. Consequently, irrational changes of individual coefficients against the trend of technology and market forces are avoided which are observed when the RAS procedure with given row and column totals is applied. Innovation, technological trends and market forces and not the enforcement of consistency have priority in the new update procedure" (Eurostat, 2008, p. 463);
10. "The Euro method has some familiarity with a price model in which a unit multiplication of rows with prices indexes would reflect the growth of primary inputs. However, Euro reflects also the substitution which is induced by the change of relative prices" (Eurostat, 2008, p. 463).

[1] Since SUTs are more fundamental data than IOTs (the latter are obtained from SUTs under different transformation assumptions), in this paper I will focus on the SUT-Euro method only, which is simply referred to as the Euro method. Nevertheless, all the results of this paper are also valid with respect to the traditional Euro method. It should be noted that the first complete mathematical presentation of the SUT-Euro method as proposed and demonstrated in Beutel (2008) appeared in Temurshoev et al. (2011).

Compared to the original source, enumeration is used in the above excerpts for convenience of later discussions. I will refer to this as "the Euro advantages list". It is apparently believed that all these advantages also characterize the method's extension to SUTs updating (with minor changes of points 4 and 6 above as applied to SUTs rather than IOTs).

This paper takes a critical stance on the Euro approach. In particular, by digging deeper into the variants of the Euro method analytically (for the first time since its introduction) this study spells out the existing and potential problems of the method, highlighting its largely ad-hoc nature. The derived analytical results make explicit the restrictive usability of and the strong or unrealistic nature of the (hidden) assumptions underlying the Euro update procedure. Some of these assumptions are further tested empirically and are found to be at odds with the data.

In addition to these main results, this paper also questions some of the claims made in Valderas-Jaramillo et al. (2019, henceforth, VROB). In particular, their belief of "unfair comparisons and misleading conclusions" made in TT is reconsidered. It is shown that even under the "fair comparison" environment (as understood by VROB) a proposed simple variant of *generalized RAS* (GRAS) outperforms the Euro method.[2]

In the following sections different empirical assessments will be made, which are based on two sets of time series of annual SUTs of Spain, available from the official website of Instituto Nacional de Estadística (`www.ine.es`): the first set of annual SUTs covers 2010-2015, with Base year of 2010, and includes 63 products (CPA 2008) and 63 industries (NACE Rev 2); the second set of SUTs spans a longer period of 2000 to 2007, with the Base year of 2000, and consists of 118 products (CNPA-96) and 75 industries (CNAE-93). Since the Euro method only applies to symmetric SUTs, the latter time-series was aggregated to 73 products and 73 industries. Final demand matrices were not aggregated, thus both series include 6 final demand categories (final consumption expenditure by households, non-profit institutions serving households, and government, gross fixed capital formation, changes in inventories, and exports).

The rest of the paper is organized as follows. Section 2 analyzes both variants of the Euro method with and without available industry outputs. The analytical examination of the methods makes explicit the underlying problems, ad-hocness of the update and the nature of adjustments, some of which are tested empirically. Section 3 examines the main VROB claim on unquestionable usefulness of the Euro method in the absence of industry output, while the issue of (un)fairness is discussed in Section 4. Concluding remarks are given is Section 5.

[2] See Lahr and de Mesnard (2004) for details of the RAS-type methods, including its history. The most flexible RAS-type framework is the so-called KRAS (K for *Konfliktfreies*) of Lenzen et al. (2009), which generalizes the GRAS method to: (i) incorporate constraints on arbitrary subsets of matrix elements, including cases of constraints' coefficients being different from 1 or -1, (ii) include reliability of the initial estimate and the external constraints, and (iii) find a compromise solution between conflicting constraints.

## 2 Digging deeper into the Euro method

Generally speaking, no empirical exercise, however extensive, can be a substitute for an indepth analytical analysis - if the latter is a viable option - in understanding the nature and features of any mathematical method. Unfortunately, the advocates of the Euro approach never presented an analytical study of the method's properties. In this section, I try to accomplish this task, at least partially. This sheds light on the (existing and potential) problems and ad-hocness of the Euro approach.

The individual components of SUTs and other variables used in the Euro method are denoted as follows:[3]

- $\mathbf{S}^d$ is the (domestic) Supply matrix of dimension product by industry ($p \times s$), which is a transpose of the Make matrix often denoted by $\mathbf{V}$;
- $\mathbf{m}$ is the $p$-dimensional vector of total imports priced at CIF, and economy-wide total imports are given by $m = \boldsymbol{\imath}'\mathbf{m}$;
- $\mathbf{U}^d$ and $\mathbf{U}^m$ are $p \times s$ matrices of, respectively, domestic and imported intermediate uses at basic prices;
- $\mathbf{Y}^d$ and $\mathbf{Y}^m$ are, respectively, $p\times f$ matrices of domestic and imported final uses at basic prices (where $f$ is the number of final use categories);
- $\mathbf{n}_u$ and $\mathbf{n}_y$ are total taxes less subsidies (or net taxes) on products (TLS) for, respectively, intermediate and final uses, whose national total is $n = [\mathbf{n}_u', \mathbf{n}_y']\boldsymbol{\imath}$;
- $\mathbf{v}$ and $\mathbf{y}$ are, respectively, the vectors of gross value added (GVA) by industry at basic prices and of totals of final demand categories at purchasers' prices;
- $\mathbf{D}_0 = \mathbf{V}_0\hat{\mathbf{q}}^{-1}$ is the base-year domestic market share matrix indicating the commodity output proportions of each industry ($\mathbf{q}$ is domestic output by product), i.e. $\boldsymbol{\imath}'\mathbf{D}_0 = \boldsymbol{\imath}'$.

In what follows, for space consideration I refer to all the variables whose future value is known under the Euro method setting as official macroeconomic forecasts (OMFs) or OMF variables. These include industry GVA $\mathbf{v}$, totals of final demand categories at purchasers' prices $\mathbf{y}$, economy-wide totals of TLS $n$, and of imports $m$. Note that the equivalence of the income and expenditure approaches to GDP measurement implies that $\boldsymbol{\imath}'\mathbf{v} + n = \boldsymbol{\imath}'\mathbf{y} - m$. For simplicity, the intermediate Use and final Use matrices are denoted, respectively, as:

$$\mathbf{U} = \begin{bmatrix} \mathbf{U}^d \\ \mathbf{U}^m \\ \mathbf{n}_u' \end{bmatrix} \quad \text{and} \quad \mathbf{Y} = \begin{bmatrix} \mathbf{Y}^d \\ \mathbf{Y}^m \\ \mathbf{n}_y' \end{bmatrix}.$$

[3]Matrices are given in bold, capitals; vectors in bold, lower cases; and scalars in italicized, lower case letters. Vectors are columns by definition, row vectors are obtained by transposition, indicated by a prime. $\hat{\mathbf{x}}$ denotes a diagonal matrix with the entries of $\mathbf{x}$ on its main diagonal and zeros elsewhere. The symbol $\circ$ indicates Hadamard product or the element-wise multiplication of matrices. The summation vector of ones of appropriate dimension is denoted by $\boldsymbol{\imath}$. Finally, subscript zero indicates that the variable in question refers to the base (or benchmark) year, whose value is thus known/observed.

## 2.1 The standard Euro method with unknown industry output

Iteration $t$ of the Euro algorithm can be compactly written as follows:[4]

$$\boldsymbol{\Delta}_t = \begin{bmatrix} \boldsymbol{\Delta}_t^v \\ \boldsymbol{\Delta}_t^y \\ \Delta_t^n \\ \Delta_t^m \end{bmatrix} = \begin{bmatrix} \hat{\mathbf{v}}_{t-1}^{-1}\mathbf{v} - \boldsymbol{\imath} \\ \hat{\mathbf{y}}_{t-1}^{-1}\mathbf{y} - \boldsymbol{\imath} \\ \frac{n}{n_{t-1}} - 1 \\ \frac{m}{m_{t-1}} - 1 \end{bmatrix} \tag{1.1}$$

Deviation rates of, respectively, GVA by industry, totals of final demand categories, economy-wide totals of TLS and imports.

$$\mathbf{c}_t = \boldsymbol{\imath} + sign(\boldsymbol{\Delta}_t)\frac{(|\boldsymbol{\Delta}_t| \times 100)^{\epsilon}}{100} \tag{1.2}$$

Correction factors, where $\epsilon > 0$ is the adjustment elasticity, often set at $\epsilon = 0.9$. For $t = 1$, all these factors are set to unity.

$$\mathbf{r}_t^h = \mathbf{c}_t^h \circ \mathbf{r}_{t-1}^h,\ r_t^n = c_t^n r_{t-1}^n,\ \mathbf{r}_t^m = c_t^m \mathbf{r}_{t-1}^m \tag{1.3}$$

Multipliers, where $h = v, y$. For $t = 1$, set $\mathbf{r}_1^v = \hat{\mathbf{v}}_0^{-1}\mathbf{v}$, $\mathbf{r}_1^y = \hat{\mathbf{y}}_0^{-1}\mathbf{y}$, $r_1^n = n/n_0$ and $\mathbf{r}_1^m = \mathbf{r}_1^v$.

$$\mathbf{Y}_t = Mean\left\{\hat{\mathbf{r}}_t\mathbf{Y}_0,\ \mathbf{Y}_0\hat{\mathbf{r}}_t^y\right\} \tag{1.4}$$

Final demand estimate, using either an arithmetic or geometric mean. Here, $\mathbf{r}_t = [(\mathbf{r}_t^v)', (\mathbf{r}_t^m)', r_t^n]'$.

$$\mathbf{U}_t^* = Mean\left\{\hat{\mathbf{r}}_t\mathbf{U}_0,\ \mathbf{U}_0\hat{\mathbf{r}}_t^v\right\} \tag{1.5}$$

Preliminary intermediate use matrix.

$$\mathbf{x}_t^* = (\mathbf{U}_t^*)'\boldsymbol{\imath} + \hat{\mathbf{r}}_t^v\mathbf{v}_0 \tag{1.6}$$

Preliminary gross output vector.

$$\mathbf{B}_t = \mathbf{U}_t^*(\hat{\mathbf{x}}_t^*)^{-1} \tag{1.7}$$

Inputs (domestic, imported and net taxes) structure.

$$\mathbf{x}_t = \left(\mathbf{I} - \mathbf{D}_0\mathbf{B}_t^d\right)^{-1}\mathbf{D}_0\mathbf{Y}_t^d\boldsymbol{\imath} \tag{1.8}$$

Gross output estimate, where $\mathbf{B}_t^d$ is the domestic input structure part of $\mathbf{B}_t$.

$$\mathbf{U}_t = \mathbf{B}_t\hat{\mathbf{x}}_t \tag{1.9}$$

Intermediate use (incl. TLS) matrix estimate.

$$\mathbf{v}_t = \mathbf{x}_t - (\mathbf{U}_t)'\boldsymbol{\imath} \tag{1.10}$$

GVA estimate, taken as a residual.

$$\mathbf{y}_t = (\mathbf{Y}_t)'\boldsymbol{\imath} \tag{1.11}$$

Final demand totals estimate.

$$n_t = [\mathbf{n}_{u,t}', \mathbf{n}_{y,t}']\boldsymbol{\imath} \tag{1.12}$$

Total TLS estimate, where $[\mathbf{n}_{u,t}', \mathbf{n}_{y,t}']$ is the last row of $[\mathbf{U}_t, \mathbf{Y}_t]$.

$$\mathbf{q}_t = \mathbf{U}_t^d\boldsymbol{\imath} + \mathbf{Y}_t^d\boldsymbol{\imath} \tag{1.13}$$

Commodity output estimate.

$$\mathbf{V}_t = \mathbf{D}_0\hat{\mathbf{q}}_t \tag{1.14}$$

Make matrix estimate.

$$\mathbf{m}_t = \mathbf{U}_t^m\boldsymbol{\imath} + \mathbf{Y}_t^m\boldsymbol{\imath} \tag{1.15}$$

Imports by product, which sums to $m_t$.

[4]This approach is analytically easier to comprehend as compared to the usual (largely) verbal description of the method, including the use of illustrations of hypothetical worked examples and flowcharts.

These 15 equations, constituting one iteration of the Euro method, are repeatedly used until its convergence, i.e. until the OMF variables are reproduced by the method. This practically means that the algorithm terminates (if feasible) whenever it reaches sufficiently small deviation rates in (1.1).

I first look into the nature of multipliers in (1.3). Without loss of generality, assume that the adjustment elasticity used in deriving the multipliers' correction factors (1.2) is equal to unity, i.e. assume $\epsilon = 1$. Then the correction factors at iteration $t$ in (1.2) boil down to the ratios of OMFs to their corresponding values obtained from the Euro algorithm at the previous iteration, i.e. $\mathbf{c}_t^v = \hat{\mathbf{v}}_{t-1}^{-1}\mathbf{v}$, $\mathbf{c}_t^y = \hat{\mathbf{y}}_{t-1}^{-1}\mathbf{y}$, $c_t^n = n/n_{t-1}$ and $c_t^m = m/m_{t-1}$. Thus iteration $t$ multipliers in (1.3) can be equivalently written as:

$$\mathbf{r}_t^v = (\hat{\mathbf{v}}_{t-1}^{-1}\hat{\mathbf{v}})(\hat{\mathbf{v}}_{t-2}^{-1}\hat{\mathbf{v}})\cdots(\hat{\mathbf{v}}_2^{-1}\hat{\mathbf{v}})(\hat{\mathbf{v}}_1^{-1}\hat{\mathbf{v}})(\hat{\mathbf{v}}_0^{-1}\mathbf{v}) \tag{2.1}$$

$$\mathbf{r}_t^y = (\hat{\mathbf{y}}_{t-1}^{-1}\hat{\mathbf{y}})(\hat{\mathbf{y}}_{t-2}^{-1}\hat{\mathbf{y}})\cdots(\hat{\mathbf{y}}_2^{-1}\hat{\mathbf{y}})(\hat{\mathbf{y}}_1^{-1}\hat{\mathbf{y}})(\hat{\mathbf{y}}_0^{-1}\mathbf{y}) \tag{2.2}$$

$$r_t^n = \left(\frac{n}{n_{t-1}}\right)\left(\frac{n}{n_{t-2}}\right)\cdots\left(\frac{n}{n_2}\right)\left(\frac{n}{n_1}\right)\left(\frac{n}{n_0}\right) \tag{2.3}$$

$$\mathbf{r}_t^m = \left(\frac{m}{m_{t-1}}\right)\left(\frac{m}{m_{t-2}}\right)\cdots\left(\frac{m}{m_2}\right)\left(\frac{m}{m_1}\right)(\hat{\mathbf{v}}_0^{-1}\mathbf{v}) \tag{2.4}$$

The form of multiplier adjustment in (2.1)-(2.4) is exactly the same as in the standard RAS approach (see e.g. Miller and Blair, 2009, pp. 316-320). The logic of such adjustments remains valid also when $\epsilon \neq 1$. Note from (2.4) that the row multipliers of imported intermediate and final uses, $\mathbf{r}_t^m$, are exactly proportional (up to a scalar) to the ratios of OMF-to-benchmark industry GVA, i.e.

$$\mathbf{r}_t^m = \mu_t\,\hat{\mathbf{v}}_0^{-1}\mathbf{v} \quad \text{where} \;\; \mu_t = c_t^m c_{t-1}^m \cdots c_2^m > 0. \tag{3}$$

Note that relation (3) holds true in general, i.e. also for $\epsilon \neq 1$. This is the first element of the Euro method's *ad-hocness*. There is no reason to expect that imports of a product will have the same growth rate as that of GVA of an industry which domestically makes the product in question. To be clearer, there might be a logic in assuming the same growth of the mix of imports by sector and that sector's GVA, but not so at the individual product level. Using Spanish SUTs, a simple Pearson's linear correlation analysis of the growth rates of commodity imports and industry GVA was carried out. The obtained results are summarized in Table 1. The 43 reported correlation coefficients range from -0.146 to 0.520, with the overall average of only 0.131. Statistically significant but small positive correlations of the growth rates are observed for the 2000-2007 data with only two benchmark years (in computing the relevant growth rates) of 2000 and 2001. All in all, from these results one can safely conclude that there is no practical basis for defining imports row multipliers to be proportional to GVA growth rates, as is assumed in (3).

It must be noted that this critical issue is acknowledged in VROB who state that "This feature of the SUT-EURO methods clearly opens the door for future improvements

**Table 1:** Correlation of the growth rates of product imports and industry GVA, Spain

| Base year | 2011 | 2012 | 2013 | 2014 | 2015 | Average | | |
|---|---|---|---|---|---|---|---|---|
| 2010 | 0.197 | 0.169 | 0.002 | -0.039 | -0.031 | 0.059 | | |
| | (0.145) | (0.213) | (0.989) | (0.774) | (0.818) | | | |
| 2011 | | -0.132 | -0.146 | -0.029 | 0.083 | -0.056 | | |
| | | (0.333) | (0.282) | (0.832) | (0.542) | | | |
| 2012 | | | 0.044 | 0.178 | 0.184 | 0.135 | | |
| | | | (0.749) | (0.191) | (0.174) | | | |
| 2013 | | | | 0.065 | -0.074 | -0.004 | | |
| | | | | (0.633) | (0.587) | | | |
| 2014 | | | | | -0.129 | -0.129 | | |
| | | | | | (0.343) | | | |
| **Base year** | **2001** | **2002** | **2003** | **2004** | **2005** | **2006** | **2007** | **Average** |
| 2000 | 0.149 | 0.520*** | 0.306** | 0.377*** | 0.401*** | 0.389*** | 0.468*** | 0.373 |
| | (0.269) | (0.000) | (0.020) | (0.004) | (0.002) | (0.003) | (0.000) | |
| 2001 | | 0.386*** | 0.167 | 0.308** | 0.260* | 0.243* | 0.329** | 0.282 |
| | | (0.003) | (0.213) | (0.020) | (0.051) | (0.068) | (0.013) | |
| 2002 | | | -0.037 | 0.100 | 0.080 | 0.084 | 0.155 | 0.076 |
| | | | (0.784) | (0.458) | (0.555) | (0.535) | (0.249) | |
| 2003 | | | | 0.088 | -0.016 | 0.020 | 0.134 | 0.057 |
| | | | | (0.514) | (0.904) | (0.880) | (0.320) | |
| 2004 | | | | | -0.125 | 0.015 | 0.155 | 0.015 |
| | | | | | (0.354) | (0.913) | (0.248) | |
| 2005 | | | | | | 0.134 | 0.192 | 0.163 |
| | | | | | | (0.319) | (0.153) | |
| 2006 | | | | | | | 0.018 | 0.018 |
| | | | | | | | (0.893) | |

*Note*: This table presents Pearson's linear correlation coefficients and the *p*-values (in parentheses) for testing the hypothesis of no correlation against the alternative of a non-zero correlation. Coefficients with ***, ** and * are significant at 1, 5 and 10% levels, respectively. The number of observations of each imports and GVA vectors were, respectively, 56 and 57 for the 2010-2015 and 2000-2007 data (i.e. respectively 7 and 16 observations corresponding to zero imports were excluded).

through the use of exogenous information on growth rates of imports (maybe from official trade statistics) instead of GVA growth rates" (p. 441). Although a welcome plan, this would then mean that: (a) the Euro method loses one of its advantages of being based on "limited data requirements" (point 2 in the Euro advantages list in the Introduction), (b) the method would use more than "only official sources" for the update (see point 3 in the list), and (c) the use of other (better-performing) updating approaches such as the GRAS method (see e.g. Günlük-Şenesen and Bates, 1988; Junius and Oosterhaven, 2003; Temurshoev et al., 2013) becomes more appealing.

The second element of ad-hocness is related to how in general multipliers in (1.3) or in (2.1)-(2.4) are defined. In the first iteration, the Euro method defines the multipliers as $\mathbf{r}_1^v = \mathbf{r}_1^m = \hat{\mathbf{v}}_0^{-1}\mathbf{v}$, $\mathbf{r}_1^y = \hat{\mathbf{y}}_0^{-1}\mathbf{y}$ and $r_1^n = n/n_0$, which evidently affects their values in later iterations as well. The problem arises when the signs of the OMF variables change compared to those of the benchmark year. This can easily happen with the final demand category of changes in inventories, but also with respect to GVA and theoretically is also a possibility with total TLS.[5] Especially if one deals with relatively large-scale SUTs (which is the future, if not already now for some countries), it is quite possible that the GVA of a sector is positive in one year but negative in the other year due to negative net operating surplus and/or other net taxes on production. However, with negative multipliers the Euro method would either break down (which is definitely the case when a geometric mean option is used) or would produce unsatisfactory results such as switching the signs of the elements of the updated Use table compared to the corresponding original base-year entries. As a result, one *cannot* claim that the Euro method is a "robust update procedure" (point 1 in the Euro advantages list).

It may also happen that TLS, changes in inventories and/or GVA of certain industries sum up to zero in one year but become non-zero in other years. In these cases the relevant multipliers in (2.1)-(2.4) either become zero or remain undefined. In both cases some other manual, ad-hoc changes need to be introduced to "solve" the problem, which again implies that the method is generally not a robust updating procedure.

It needs to be stressed that the possibility of changing the signs of an IOT's row and column totals was not an issue in early days of RAS applications, as it was then meant to be used exclusively for updating input coefficients matrix which, by definition, is non-negative. This is part of the reason of the claimed fourth advantage of the Euro method as "integrated estimation of all four quadrants of the IOT". However, nowadays biproportional adjustment is applied to all parts (if necessary) of an IOT/SUT or any other matrix using the GRAS approach. This explicitly accounts for the possibility of sign-switching, zero-to-nonzero or nonzero-to-zero switching constraints' totals in a theoretically rigorous way. In addition, most of the existing updating methods can also be applied for integrated estimation of all parts of an IOT/SUT (see e.g. TWY). Hence point 4 of the Euro advantages list is not Euro-specific but is equally valid for most IOT/SUT updating methods.

Now observe that (1.7) and (1.9) together imply:

$$\mathbf{U}_t = \mathbf{B}_t \hat{\mathbf{x}}_t = \mathbf{U}_t^* (\hat{\mathbf{x}}_t^*)^{-1} \hat{\mathbf{x}}_t = \mathbf{U}_t^* \hat{\mathbf{r}}_t^x, \tag{4}$$

where $\mathbf{r}_t^x \equiv (\hat{\mathbf{x}}_t^*)^{-1} \mathbf{x}_t$, which I refer to as gross output multipliers. It must be noted that if $t$ is the final iteration of the Euro algorithm, then generally it will be the case that $\mathbf{r}_t^x \neq \boldsymbol{\imath}$. That is, even after convergence the preliminary gross outputs in (1.6) will be different from the ultimate gross outputs in (1.8) that are consistent with the fixed commodity sales structure model. In fact, this will also be the case with the GVA estimate in (1.6), i.e. $\hat{\mathbf{r}}_t^v \mathbf{v}_0 \neq \mathbf{v}$. However, the following result always holds true:

[5] One can "generalize" the framework by incorporating TLS matrices $\mathbf{N}_u$ and $\mathbf{N}_y$ instead of, respectively, their column totals of $\mathbf{n}_u'$ and $\mathbf{n}_y'$ within the Use table. In such a setting, there is a higher chance of encountering cases when the signs of product-level TLS totals, $[\mathbf{N}_u, \mathbf{N}_y]\boldsymbol{\imath}$, switch across periods.

**Proposition 1** *In the Euro method, the base-year and forecast (or target) industry GVA, gross output multipliers and value-added multipliers are related as follows:* $\mathbf{v}_t = \hat{\mathbf{r}}_t^x \hat{\mathbf{r}}_t^v \mathbf{v}_0$.

*Proof*: Equations (1.10) and (4) together imply: $\mathbf{v}_t = \mathbf{x}_t - (\mathbf{U}_t)'\boldsymbol{\imath} = \mathbf{x}_t - (\mathbf{U}_t^* \hat{\mathbf{r}}_t^x)'\boldsymbol{\imath} = \mathbf{x}_t - \hat{\mathbf{r}}_t^x (\mathbf{U}_t^*)'\boldsymbol{\imath}$. Further, from (1.6) one obtains $(\mathbf{U}_t^*)'\boldsymbol{\imath} = \mathbf{x}_t^* - \hat{\mathbf{r}}_t^v \mathbf{v}_0$, which upon substitution into the last expression yields: $\mathbf{v}_t = \mathbf{x}_t - \hat{\mathbf{r}}_t^x(\mathbf{x}_t^* - \hat{\mathbf{r}}_t^v \mathbf{v}_0) = \hat{\mathbf{r}}_t^x \hat{\mathbf{r}}_t^v \mathbf{v}_0$. □

After convergence it must be the case that $\mathbf{v}_t = \mathbf{v}$, which in practice is an approximate equality. Thus, assuming that convergence is achieved at iteration $t$, Proposition 1 implies that the genuine GVA multipliers, i.e. those that convert $\mathbf{v}_0$ to $\mathbf{v}$, are the elements of the vector $\mathbf{r}^{GVA} \equiv \hat{\mathbf{r}}_t^x \mathbf{r}_t^v$ and not $\mathbf{r}_t^v$ alone.[6] This is because of the Euro method's additional necessary adjustments of converting the preliminary outputs in (1.6) to those in (1.8) in order to guarantee the equality of total outputs and total inputs by industry. Since both $\mathbf{v}_0$ and $\mathbf{v}$ are known, it is then also the case that $r_i^x r_i^v = 1 + g_i^v$ for each industry $i$, where $g_i^v$ is $i$-th GVA growth rate.[7]

Equations (1.4), (1.5) and (4) can be used jointly to spell out the exact nature of the Euro adjustments of the Use table components, which in turn determine the system supply side variables according to (1.13), (1.14) and (1.15). The following result reveals the Euro adjustments nature for both cases when the Use matrices are obtained using arithmetic and geometric mean options.[8]

**Proposition 2** *The six unknown components of the Use table according to the Euro method are adjusted/updated in the following way.*

- *When the Use table update is based on the arithmetic mean option:*

$$\mathbf{Y}^d = \mathbf{A}_Y^d \circ \mathbf{Y}_0^d \quad \textit{with } (\mathbf{A}_Y^d)_{if} = \left(r_i^v + r_f^y\right)/2 \tag{5.1}$$

$$\mathbf{Y}^m = \mathbf{A}_Y^m \circ \mathbf{Y}_0^m \quad \textit{with } (\mathbf{A}_Y^m)_{if} = \left(\mu\, v_i / v_{0,i} + r_f^y\right)/2 \tag{5.2}$$

$$\mathbf{n}_y = \hat{\mathbf{a}}_Y^n \mathbf{n}_{y0} \quad \textit{with } (\mathbf{a}_Y^n)_f = \left(r^n + r_f^y\right)/2 \tag{5.3}$$

$$\mathbf{U}^d = \mathbf{A}_U^d \circ \mathbf{U}_0^d \quad \textit{with } (\mathbf{A}_U^d)_{ij} = \frac{1}{2}\left(1 + \frac{r_i^v}{r_j^v}\right)\frac{v_j}{v_{0,j}} \tag{5.4}$$

$$\mathbf{U}^m = \mathbf{A}_U^m \circ \mathbf{U}_0^m \quad \textit{with } (\mathbf{A}_U^m)_{ij} = \frac{1}{2}\left(1 + \frac{\mu\, v_i}{v_{0,i}\, r_j^v}\right)\frac{v_j}{v_{0,j}} \tag{5.5}$$

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[6] The expression in Proposition 1 also reveals the following relation between the preliminary and final estimate of gross output at each iteration: $\mathbf{x}_t = (\hat{\mathbf{r}}_t^v \hat{\mathbf{v}}_0)^{-1} \hat{\mathbf{v}}_t \mathbf{x}_t^*$.

[7] Sometimes when a reference is made to the ultimate (i.e. after-convergence) multipliers and variables, their iteration identifier $t$ is dropped for convenience.

[8] A point of clarification with respect to the use of the geometric mean option is in order here. For negative entries of SUTs (such as changes in inventories or net taxes on products), using the standard geometric mean formula is insufficient, if one opts to use the known algorithm of the Euro method, (1.1)-(1.15). Then to keep the negative sign, the latter has to be added manually. Thus, in practice equation (1.4) with geometric mean option needs to be implemented as: $\mathbf{Y}_t = sign(\mathbf{Y}_0) \circ \sqrt{\left(\hat{\mathbf{r}}_t \mathbf{Y}_0\right) \circ \left(\mathbf{Y}_0 \hat{\mathbf{r}}_t^y\right)}$. However, such sign adjustment is not necessary with the alternative formulation of the Euro method presented in Proposition 2, provided that all the multipliers are positive.

$$\mathbf{n}_u = \hat{\mathbf{a}}_U^n \, \mathbf{n}_{u0} \quad with \; (\mathbf{a}_U^n)_j = \frac{1}{2}\left(1 + \frac{r^n}{r_j^v}\right)\frac{v_j}{v_{0,j}} \tag{5.6}$$

*where the corresponding adjustment factors' typical entry identifiers i, j and f refer to product i, industry j and final demand category f, respectively.*

- *When the Use table update is based on the geometric mean option:*

$$\mathbf{Y}^d = \sqrt{\hat{\mathbf{r}}^v}\, \mathbf{Y}_0^d \sqrt{\hat{\mathbf{r}}^y} \;\; = \hat{\mathbf{r}}_v \, \mathbf{Y}_0^d \, \hat{\mathbf{s}}_y \tag{6.1}$$

$$\mathbf{Y}^m = \sqrt{\mu}\sqrt{\hat{\mathbf{v}}_0^{-1}\hat{\mathbf{v}}}\; \mathbf{Y}_0^m \sqrt{\hat{\mathbf{r}}^y} \;\; = \hat{\mathbf{r}}_m \, \mathbf{Y}_0^m \, \hat{\mathbf{s}}_y \tag{6.2}$$

$$\mathbf{n}'_y = \sqrt{r^n}\, \mathbf{n}'_{y0} \sqrt{\hat{\mathbf{r}}^y} \;\; = r_n \, \mathbf{n}'_{y0} \, \hat{\mathbf{s}}_y \tag{6.3}$$

$$\mathbf{U}^d = \sqrt{\hat{\mathbf{r}}^v}\, \mathbf{U}_0^d \, \hat{\mathbf{v}}_0^{-1}\hat{\mathbf{v}}\left(\sqrt{\hat{\mathbf{r}}^v}\right)^{-1} \;\; = \hat{\mathbf{r}}_v \, \mathbf{U}_0^d \, \hat{\mathbf{s}}_u \tag{6.4}$$

$$\mathbf{U}^m = \sqrt{\mu}\sqrt{\hat{\mathbf{v}}_0^{-1}\hat{\mathbf{v}}}\; \mathbf{U}_0^m \, \hat{\mathbf{v}}_0^{-1}\hat{\mathbf{v}}\left(\sqrt{\hat{\mathbf{r}}^v}\right)^{-1} \;\; = \hat{\mathbf{r}}_m \, \mathbf{U}_0^m \, \hat{\mathbf{s}}_u \tag{6.5}$$

$$\mathbf{n}'_u = \sqrt{r^n}\, \mathbf{n}'_{u0}\hat{\mathbf{v}}_0^{-1}\hat{\mathbf{v}}\left(\sqrt{\hat{\mathbf{r}}^v}\right)^{-1} \;\; = r_n \, \mathbf{n}'_{u0} \, \hat{\mathbf{s}}_u. \tag{6.6}$$

*For simplicity, in presenting the expressions (5.1)-(6.6) the iteration identifier t is suppressed and no further notational distinction is made for the same multiplier across the two Euro variants (e.g. $r^n$ will have one value in (5.3) and (5.6), but a different value in (6.3) and (6.6)).*

*Proof*: The proof is skipped as it readily follows from the definitions of arithmetic and geometric means, equations (1.4), (1.5), (3), (4), and Proposition 1 which implies that output multipliers can be replaced by $\hat{\mathbf{r}}_t^x = (\hat{\mathbf{r}}_t^v \hat{\mathbf{v}}_0)^{-1}\hat{\mathbf{v}}_t$. □

The important implications of Proposition 2 are as follows. First, the arithmetic mean-based Euro method (Euro-A) updates each element of the Use table applying *cell-specific adjustment factors*, in some sense akin to the approach of Harthoorn and van Dalen (1987). The latter method, however, is more general/flexible and has a sounder theoretical basis compared to the Euro method (see e.g. TWY). The Use components' adjustment factors of the Euro method have very specific form: for final uses, as seen from (5.1)-(5.3), the adjustment factors are simply arithmetic means of the relevant multipliers, while for intermediate uses such arithmetic means are further adjusted (multiplied) by gross output multipliers of the purchasing sector whose simplified expressions appear in (5.4)-(5.6). Such specification of the adjustment factors is admittedly quite *restrictive* since e.g. the relative importance (or weights) of the updated elements are not properly accounted for during the adjustment process.[9] I think such restrictive specification of the adjust-

[9] In this respect, the generality and flexibility of the Harthoorn and van Dalen (1987, HvD) method can be seen from the expression of its optimal cell-specific factor as $a_{ij} = 1 + g_{ij}(\lambda_i + \tau_j)/u_{0,ij}$. Here, $a_{ij}$ minimizes the objective function $\sum_i \sum_j (f_{ij}u_{0,ij} - u_{0,ij})^2/g_{ij}$ subject to the row and column sums constraints of the new Use table, $\lambda_i$ and $\tau_j$ are, respectively, the Lagrange multipliers of these constraints, and $g_{ij}$ is the exogenously specified relative confidence of the benchmark element $u_{0,ij}$. Different specifications of $g_{ij}$ are possible, such as $g_{ij} = 1$ for all $i$ and $j$, $g_{ij} = |u_{0,ij}|$ and $g_{ij} = (u_{0,ij})^2$. (Multipliers $\lambda_i$ and $\tau_j$ are also a function of $g_{ij}$.) From these three specifications (one might try other justifiable choices), the empirical evaluations of updating SUTs by TWY showed that the choice $g_{ij} = |u_{0,ij}|$ resulted in the best-performing HvD variant.

ment factors explains the general underperformance of the Euro-A method compared to the geometric mean-based Euro approach (Euro-G).[10]

Second, as follows from (6.1)-(6.6), the Euro-G method updates the Use table employing the traditional RAS adjustment approach. In fact, assume one knows in advance the Euro-G's commodity outputs $\mathbf{q}$ or industry outputs $\mathbf{x}$ (as the other missing output vector can be obtained from $\mathbf{x} = \mathbf{D}_0\mathbf{q}$ as follows from (1.14)) and the imports vector $\mathbf{m}$, and encounters the base matrix $[\mathbf{U}_0, \mathbf{Y}_0]$ *without* any negative entry. Then RASing the latter benchmark matrix to satisfy the new row and column sums of, respectively, $[\mathbf{q}', \mathbf{m}', n]'$ and $[\mathbf{x}' - \mathbf{v}', \mathbf{y}']'$ results in $[\mathbf{U}, \mathbf{Y}]$ that is *exactly equivalent* to that of the Euro-G method. This should not be surprising, as de Mesnard (1994) already rigorously showed that *any* algorithm used to compute a *biproportion* of a matrix with real positive numbers leads to the same result. This outcome and the general superiority of the Euro-G over the Euro-A approach render baseless any claim on the superiority of the Euro over the (G)RAS method, such as the statement that the Euro method "avoids arbitrary changes of important input coefficients, which sometimes occur if traditional RAS procedures are applied" (Eurostat, 2008, p. 461) or the 9-th point in the Euro advantages list. This is because *the Euro-G is nothing else but an RAS variant*, which in comparison to the standard RAS additionally endogenously derives the vectors of domestic and imported products. As a matter of fact, one can easily think of other RAS variants, where these vectors are also endogenously obtained. The issues raised so far and to be raised in what follows with respect to the Euro-G variant also imply that *Euro-G is a restricted version of RAS*. Therefore, by construction, (G)RAS can still outperform Euro if: (a) different missing vectors $\mathbf{q}$, $\mathbf{x}$ and $\mathbf{m}$ are used that are closer to their actual counterparts, or (b) other GRAS variants are employed (a simple one to be presented in Section 3).

Third, in the two Euro variants negative elements of SUTs are adjusted in the same way as positive elements. This approach is an ad-hoc treatment of negatives, which was already convincingly argued in Junius and Oosterhaven (2003) with respect to traditional RAS. In particular, equal treatment of positive and negative entries may easily produce a new updated SUT that strongly deviates from the structure of its benchmark table. "This is especially likely in the rows and columns of the relatively larger negative entries. In these rows and columns, the positive entries may have to be adjusted far more than the positive cells in the other rows and columns, as the negative entries have a negative contribution to the adjustment in each iterative round" (Junius and Oosterhaven, 2003, p. 88). The empirical confirmation of this crucial issue is provided in Section 3.

Fourth, similar to the first point regarding the Euro-A method, the nature of biproportional adjustment under the Euro-G is quite restrictive. Assume the algorithm con-

[10]From their empirical assessments, VROB conclude that "the geometric version of the SUT-EURO method usually performs better than its arithmetic version" (p. 442).

verges, hence $\mathbf{v}_t = \mathbf{v}$ for all practical purposes.[11] Then as follows from (6.4) and (6.5) the final estimates of a typical element of the domestic and imported intermediate Use table take, respectively, the following forms:

$$u_{ij}^d = \sqrt{r_i^v}\left(u_{0,ij}^d\right)\frac{v_j}{v_{0,j}}\frac{1}{\sqrt{r_j^v}}\,, \tag{7.1}$$

$$u_{ij}^m = \sqrt{\mu}\sqrt{\frac{v_i}{v_{0,i}}}\left(u_{0,ij}^m\right)\frac{v_j}{v_{0,j}}\frac{1}{\sqrt{r_j^v}}\,. \tag{7.2}$$

Looking closer into the multipliers in (7.1) and (7.2) and those of the intermediate and final demand matrices under both Euro variants given in Proposition 2 clarifies the last two points in the Euro advantages list. It is apparently understood that "innovative sectors" (resp. "declining sectors") are those sectors whose value added has increased (resp. decreased) compared to the benchmark year. That is, the presence of the term $v_j/v_{0,j}$ in the multipliers of both the Euro-G and Euro-A variants "guarantees that innovative sectors gain in relative importance in all activities, while declining sectors lose in importance everywhere. Consequently, irrational changes of individual coefficients against the trend of technology and market forces are avoided..." (point 9 of the Euro advantages list). Similarly, the statement that "Euro reflects also the substitution which is induced by the change of relative prices" (point 10 in the Euro advantages list) can be understood that such substitution effects are captured by multipliers $\sqrt{r_i^v/r_j^v}$ and $\sqrt{(\mu v_i/v_{0,i})/r_j^v}$ in cases of sectors' purchases of, respectively, domestically produced and imported goods. This implies that the *domestic price index* for product $i$ is captured by $\sqrt{r_i^v}$, while the *import price index* for product $i$ is approximated by $\sqrt{\mu v_i/v_{0,i}}$.[12] The latter finding explicitly explains the first part of the last point in the Euro advantages list: "The Euro method has some familiarity with a price model in which a unit multiplication of rows with prices indexes would reflect the growth of primary inputs." That is, since the aim of the Euro approach was updating SUT/IOT with limited data requirements, the reasonable approximation of import prices was to assume that these are proportional to the growth of GVA. This is the same weakness of the Euro approach that was discussed earlier on the correlation between total imports and GVA growth rates. In terms of relative price interpretation, it

[11] This applies to the intermediate Use table *column* multipliers, but not imports row multipliers which by construction are proportional to the OMF-to-benchmark GVA ratio, (3). For example, the iteration $t$ imported intermediate Use matrix in (6.5) with explicit $t$ can be written as $\mathbf{U}_t^m = \sqrt{\mu_t}\sqrt{\hat{\mathbf{v}}_0^{-1}\hat{\mathbf{v}}}\,\mathbf{U}_0^m\,\hat{\mathbf{v}}_0^{-1}\hat{\mathbf{v}}_t\left(\sqrt{\hat{\mathbf{r}}_t^v}\right)^{-1}$.

[12] These last four expressions, but without square root, would also capture - within the corresponding arithmetic mean terms - the effects of substitution and price indexes under the Euro-A approach. E.g., the term $0.5(1 + r_i^v/r_j^v)$ in (5.4) may be interpreted as the relative price index relevant to sector $j$'s purchases of domestic intermediates per unit of its output. The two terms in this average include the base-year unitary relative price of products $i$ to $j$ and the corresponding relative price for the forecast year $r_i^v/r_j^v$.

is highly likely that in reality import prices are not proportional to GVA growth rates, especially in the globalized world with extensive cross-country production fragmentation.

An alternative interpretation of the adjustment factors of intermediate Use elements could be also given. For example, (7.1) is equivalent to $u^d_{ij} = (r^v_i)^{1/2} u^d_{0,ij} (r^v_j)^{1/2}\, r^x_j$, wherein the gross output multiplier of industry $j$, $r^x_j = v_j/(r^v_j v_{0,j})$ (see Proposition 1), would account for proper updating of transactions of innovative/declining industries, or "production effects".[13] This alternative equivalent presentation seems to be more in line with the original formulation of the last point in the Euro advantages list in Beutel (2002) as follows: "Substitution processes are changing inputs (rows), production effects are influencing outputs (columns) and price effects are affecting inputs and outputs" (p. 112).

However, the biproportional forms in (7.1) and (7.2) are obviously less flexible than a general biproportion of the form $r_i\, u^m_{0,ij}\, s_j$.[14] This follows due to the fact that the row and column multipliers in (7.1) have a one-to-one relationship at the most disaggregate industry level (as the OMF-to-benchmark GVA ratios, $v_j/v_{0,j}$, are known under the Euro updating framework), while the row multipliers in (7.2) are proportional to $v_j/v_{0,j}$ for all industries (recall earlier discussions around equation (3)). To vividly see the nature of such ad-hocness, consider the diagonal elements of $\mathbf{U}^d$ and $\mathbf{U}^m$, in which case expressions (7.1) and (7.2) simplify to:

$$u^d_{ii} = \frac{v_i}{v_{0,i}}\, u^d_{0,ii}\,, \tag{8.1}$$

$$u^m_{ii} = \sqrt{\mu} \left(\frac{v_i}{v_{0,i}}\right)^{\frac{3}{2}} \left(r^v_i\right)^{-\frac{1}{2}} u^m_{0,ii}\,. \tag{8.2}$$

It turns out that (8.1) also holds exactly valid under the Euro-A update which could be easily verified from (5.4).[15] Therefore, one is left with checking the validity of the Euro method's outcome that *the domestic intra-industry intermediate demand and GVA have exactly identical growth rates for each industry*, (8.1). Assuming that the data at hand are reliable (which is probably less certain when domestic and imported Use tables are distinguished), I again carried out a simple correlation analysis. For the 2010-2015 SUTs of Spain, separate domestic and imported Use matrices are available only for 2010 and 2015. Hence, for this dataset only one correlation coefficient between the domestic intra-industry intermediate transactions and GVA growth rates could be computed. With 62 observations, it was found to be equal to 0.235 with $p$-value of 0.066 (hence statistically

[13] The Euro-A counterpart of this expression is $u^d_{ij} = 0.5(r^v_i + r^v_j)\, u^d_{0,ij}\, r^x_j$ as follows from (5.4).

[14] Since this general biproportion holds true for updating domestic final demand matrix in (6.1), then one can expect that the Euro-G estimate of $\mathbf{Y}^d$ will have lower error compared to the estimates of other components of the Use table. This is indeed what empirical results show.

[15] As follows from (5.5), the Euro-A's counterpart of (8.2) takes the form : $u^m_{ii} = \frac{1}{2}\left[\frac{v_i}{v_{0,i}} + \frac{\mu}{r^v_i}\left(\frac{v_i}{v_{0,i}}\right)^2\right] u^m_{0,ii}$. Recall that the same multipliers (e.g. $\mu$ or $r^v_i$) differ in value across the Euro-A and Euro-G variants.

significant at the 10% level). The results for the 2000-2007 SUTs time-series, which explicitly distinguishes between domestic and imported Use tables for all covered years, are reported in Table 2. These 28 correlation coefficients average to 0.218 (comparable to 0.235 for 2015-to-2010 correlation), from which 64% are statistically significant at up to the 10% level. However, the maximum correlation was found to be 0.514. All in all, the assumption behind (8.1) is *not* confirmed by the data, as all the coefficients are far from being close to perfect positive correlation of 1.

**Table 2:** Correlation of the growth rates of intra-industry domestic intermediate transactions and GVA by industry, Spain

| Base year | 2001 | 2002 | 2003 | 2004 | 2005 | 2006 | 2007 | Average |
|---|---|---|---|---|---|---|---|---|
| 2000 | -0.008 | 0.245** | 0.108 | 0.377*** | -0.299** | 0.431*** | 0.513*** | 0.195 |
| | (0.946) | (0.038) | (0.365) | (0.001) | (0.011) | (0.000) | (0.000) | |
| 2001 | | 0.313*** | 0.286** | 0.152 | -0.003 | 0.352*** | 0.514*** | 0.269 |
| | | (0.007) | (0.015) | (0.201) | (0.979) | (0.002) | (0.000) | |
| 2002 | | | 0.237** | 0.241** | 0.306*** | 0.330*** | 0.342*** | 0.291 |
| | | | (0.045) | (0.041) | (0.009) | (0.005) | (0.003) | |
| 2003 | | | | 0.303** | 0.156 | 0.256** | 0.247** | 0.241 |
| | | | | (0.010) | (0.191) | (0.030) | (0.036) | |
| 2004 | | | | | 0.027 | 0.044 | 0.212 | 0.094 |
| | | | | | (0.824) | (0.713) | (0.074) | |
| 2005 | | | | | | 0.177 | 0.139 | 0.158 |
| | | | | | | (0.137) | (0.245) | |
| 2006 | | | | | | | 0.102 | 0.102 |
| | | | | | | | (0.395) | |

*Note*: See notes to Table 1. There were 72 observations in all 28 considered cases.

The above findings make explicit the other weakness of the Euro approach: intra-industry domestic intermediate flows are simply adjusted by the corresponding OMF-to-benchmark GVA ratios, but such updating does not have empirical support (at least, in case of Spain). However, these transactions are (or could be) important and sometimes can be quite large. For the 2010 and 2015 dataset these make up, on average per industry, about 20% of the total *domestic* intermediate demand with a maximum of up to 64%. The corresponding figures for the more disaggregated 2000-2007 dataset are roughly 12% and 59%, respectively. Note also that (8.1) explicitly shows one particular case of sign-switching intra-industry transactions under the Euro-A setting (out of many sign-switching possibilities) when industry *i*'s GVA changes its sign, i.e. when $(v_i/v_{0,i}) < 0$. Such cases, however, result in unfeasible solution of the Euro-G variant, since negative multipliers do not have real square roots (see Proposition 2).

In the same vein, one could also see an element of ad-hocness in (8.2): in updating an imported intermediate flow to a domestic industry from the same foreign industry, why is the corresponding OMF-to-benchmark GVA ratio taken to the power of 3/2 = 1.5? It is clear that this is the product of the assumptions on import price indexes and "rational" updating of purchases by the "innovative/declining sectors" discussed earlier. However, why such a peculiar adjustment for the flow in question could be considered a *universal rule* for all purchasing sectors in a particular country or for the same purchasing sectors located in different countries or for purchasing sectors with identical OMF-to-benchmark GVA ratios and identical $u_{0,ii}$'s but otherwise different? Such "rather odd-looking coefficients" without a sound theoretical and/or empirical justification should always be a reason for concerns and often are the sign of arbitrariness.[16]

Finally, the Euro method's applicability to only square SUT system is an important drawback as a rectangular SUT system with a larger number of products than industries is far more useful data for policy support and compilation of national accounts.

> "Rectangular tables do not only provide a much more detailed picture of the supply and use of goods and services in an economy. A higher degree of product detail supports also the use of certain estimation methods, such as the commodity flow method of compiling national accounts by taking into account the relevant differentiation concerning product tax rates, margin rates and homogeneity in prices... Consequently, *it is advisable to work at a level of product data with as much detail as possible*" (Eurostat, 2008, p. 44, italics added).

Allowing for finer product heterogeneity has significant implications for real-world policy when rectangular SUTs are used in impact assessments (see e.g. Lenzen and Rueda-Cantuche, 2012).

## 2.2 The Euro variant with known industry output

With their focus on "fair comparisons" of the Euro and SUR-RAS updating methods, VROB introduces a new variant of the Euro method with exogenously given industry outputs for the forecast year. In such a setting equations (1.6)-(1.9) are no longer necessary, which in the standard Euro approach are used to estimate (and employ) the missing gross output vector. To be consistent with the original Euro method, VROB propose using the base-year market share matrix $\mathbf{D}_0$ in this Euro variant as well. As a result not only gross outputs $\mathbf{x}$ are known, but also the estimates of commodity outputs and the domestic Supply (or Make) matrix are given exogenously (i.e. outside the Euro method's algorithm). They are obtained, respectively, from

$$\mathbf{q} = \mathbf{D}_0^{-1}\mathbf{x} \quad \text{and} \quad \mathbf{V} = \mathbf{D}_0\hat{\mathbf{q}}. \tag{9}$$

Thus, besides the original Euro's OMF variables $\mathbf{v}$, $\mathbf{y}$, $n$ and $m$, additionally both industry and commodity output vectors $\mathbf{x}$ and $\mathbf{q}$ are known from the outset. However, using (9) to estimate commodity output and the domestic Supply matrix is already restrictive as in reality industries' commodity output proportions are not constant.

[16] The last expression in quotes is borrowed from Kop Jansen (1994, p. 64) when the author discusses how Prof. Thijs ten Raa disliked the "rather odd-looking coefficients" of the West (1986) formulas of multiplier bias and variance, suggesting "more natural formulas".

Iteration $t$ of this Euro variant is as follows:

$$\boldsymbol{\Delta}_t = \begin{bmatrix} \boldsymbol{\Delta}_t^q \\ \boldsymbol{\Delta}_t^u \\ \boldsymbol{\Delta}_t^y \\ \Delta_t^n \\ \Delta_t^m \end{bmatrix} = \begin{bmatrix} \hat{\mathbf{q}}_{t-1}^{-1}\mathbf{q} - \boldsymbol{\imath} \\ \hat{\mathbf{u}}_{t-1}^{-1}\mathbf{u} - \boldsymbol{\imath} \\ \hat{\mathbf{y}}_{t-1}^{-1}\mathbf{y} - \boldsymbol{\imath} \\ \frac{n}{n_{t-1}} - 1 \\ \frac{m}{m_{t-1}} - 1 \end{bmatrix} \tag{10.1}$$

Deviation rates of, respectively, commodity output $\mathbf{q}$, intermediate consumption by industry at purchaser's prices $\mathbf{u}$ (where $\mathbf{u} = \mathbf{x} - \mathbf{v}$), totals of final demand categories, economy-wide totals of TLS and of imports.

$$\mathbf{c}_t = \boldsymbol{\imath} + sign(\boldsymbol{\Delta}_t)\frac{(|\boldsymbol{\Delta}_t| \times 100)^{\epsilon}}{100} \tag{10.2}$$

Correction factors. For $t = 1$, all these factors are set to unity.

$$\mathbf{r}_t^h = \mathbf{c}_t^h \circ \mathbf{r}_{t-1}^h,\ r_t^n = c_t^n r_{t-1}^n,\ \mathbf{r}_t^m = c_t^m \mathbf{r}_{t-1}^m \tag{10.3}$$

Multipliers, where $h = q, u, y$. For $t = 1$, set $\mathbf{r}_1^q = \hat{\mathbf{q}}_0^{-1}\mathbf{q}$, $\mathbf{r}_1^u = \hat{\mathbf{u}}_0^{-1}\mathbf{u}$, $\mathbf{r}_1^y = \hat{\mathbf{y}}_0^{-1}\mathbf{y}$, $r_1^n = n/n_0$ and $\mathbf{r}_1^m = \hat{\mathbf{v}}_0^{-1}\mathbf{v}$.

$$\mathbf{Y}_t = Mean\left\{\hat{\mathbf{r}}_t\mathbf{Y}_0,\ \mathbf{Y}_0\hat{\mathbf{r}}_t^y\right\} \tag{10.4}$$

Final demand estimate, using either an arithmetic or geometric mean. Here, $\mathbf{r}_t = [(\mathbf{r}_t^q)', (\mathbf{r}_t^m)', r_t^n]'$.

$$\mathbf{U}_t = Mean\left\{\hat{\mathbf{r}}_t\mathbf{U}_0,\ \mathbf{U}_0\hat{\mathbf{r}}_t^u\right\} \tag{10.5}$$

Intermediate use (incl. TLS) matrix estimate.

$$\mathbf{u}_t = (\mathbf{U}_t)'\boldsymbol{\imath} \tag{10.6}$$

Total intermediate use estimate.

In addition to (10.1)-(10.6), equations (1.11), (1.12),(1.13) and (1.15) are used to estimate, respectively, iteration $t$'s totals of final demand categories, TLS total, the vector of commodity outputs and imports vector/total. The algorithm converges whenever it reaches sufficiently small deviation rates in (10.1), at which stage the final estimates of the Use table components are obtained.

Similar to Proposition 2, the exact nature of adjustments of the Euro approach with exogenously given industry and commodity outputs (exo-Euro method) are summarized below (the proof is straightforward and is thus skipped).

**Proposition 3** *The six unknown components of the Use table according to the Euro method with exogenously given industry and commodity outputs are adjusted/updated in the following way.*

- *When the Use table update is based on the arithmetic mean option:*

$$\mathbf{Y}^d = \mathbf{A}_Y^d \circ \mathbf{Y}_0^d \quad with\ (\mathbf{A}_Y^d)_{if} = \left(r_i^q + r_f^y\right)/2 \tag{11.1}$$

$$\mathbf{Y}^m = \mathbf{A}_Y^m \circ \mathbf{Y}_0^m \quad with\ (\mathbf{A}_Y^m)_{if} = \left(\mu\, v_i/v_{0,i} + r_f^y\right)/2 \tag{11.2}$$

$$\mathbf{n}_y = \hat{\mathbf{a}}_Y^n\, \mathbf{n}_{y0} \quad with\ (\mathbf{a}_Y^n)_f = \left(r^n + r_f^y\right)/2 \tag{11.3}$$

$$\mathbf{U}^d = \mathbf{A}_U^d \circ \mathbf{U}_0^d \quad with\ (\mathbf{A}_U^d)_{ij} = \left(r_i^q + r_j^u\right)/2 \tag{11.4}$$

$$\mathbf{U}^m = \mathbf{A}_U^m \circ \mathbf{U}_0^m \quad with\ (\mathbf{A}_U^m)_{ij} = \left(\mu\, v_i/v_{0,i} + r_j^u\right)/2 \tag{11.5}$$

$$\mathbf{n}_u = \hat{\mathbf{a}}_U^n \mathbf{n}_{u0} \quad with \ (\mathbf{a}_U^n)_j = \left(r^n + r_j^u\right)/2. \tag{11.6}$$

- *When the Use table update is based on the geometric mean option:*

$$\mathbf{Y}^d = \sqrt{\hat{\mathbf{r}}^q}\, \mathbf{Y}_0^d \sqrt{\hat{\mathbf{r}}^y} \tag{12.1}$$

$$\mathbf{Y}^m = \sqrt{\mu}\sqrt{\hat{\mathbf{v}}_0^{-1}\hat{\mathbf{v}}}\, \mathbf{Y}_0^m \sqrt{\hat{\mathbf{r}}^y} \tag{12.2}$$

$$\mathbf{n}_y' = \sqrt{r^n}\, \mathbf{n}_{y0}' \sqrt{\hat{\mathbf{r}}^y} \tag{12.3}$$

$$\mathbf{U}^d = \sqrt{\hat{\mathbf{r}}^q}\, \mathbf{U}_0^d \sqrt{\hat{\mathbf{r}}^u} \tag{12.4}$$

$$\mathbf{U}^m = \sqrt{\mu}\sqrt{\hat{\mathbf{v}}_0^{-1}\hat{\mathbf{v}}}\, \mathbf{U}_0^m \sqrt{\hat{\mathbf{r}}^u} \tag{12.5}$$

$$\mathbf{n}_u' = \sqrt{r^n}\, \mathbf{n}_{u0}' \sqrt{\hat{\mathbf{r}}^u}. \tag{12.6}$$

*For simplicity, in presenting the expressions* (11.1)-(12.6) *the iteration identifier t is suppressed and no further notational distinction is made for the same multiplier across the two Euro variants.*

Given the availability of $\mathbf{q}$ and $\mathbf{u}$, the adjustments in Proposition 3 are *improved* versions of the corresponding expressions given in Proposition 2. Now the presence of two additional multipliers, $\mathbf{r}^q$ and $\mathbf{r}^u$, makes the update procedure more flexible. For example, the row and column multipliers in deriving $\mathbf{U}^d$ with the geometric mean option in (12.4) are no longer directly related at the industry level as was the case in (6.4). Nonetheless, the actual performance of the exo-Euro method in real-world updates depends on the accuracy of the estimate of commodity output $\mathbf{q}$ compared to its actual counterpart.

As with the Euro-G approach, the exo-Euro-G variant is again a restricted RAS variant, which endogenously derives the vector of total imports. This Euro-RAS link has the same consequences and implications as discussed in Section 2.1. Without much further discussion, note that most of the concerns raised earlier remain valid with respect to the exo-Euro method as well. There is still the problem of linking total imports to GVA growth rates as follows from the adjustment expressions for imported final and intermediate matrices in (11.2), (11.5), (12.2) and (12.5). In the exo-Euro-A variant the relative importance or weights of the Use table elements are not properly accounted for. There is a realistic possibility of obtaining negative multipliers with unfavorable consequences, and zero and/or undefined multipliers that would require additional ad-hoc treatment. As in the standard Euro approach, positive and negative elements are treated equally also under the exo-Euro update. This may easily lead to large structural deviations of the updated Use table compared to its original benchmark. The necessity of using the inverse of the market share matrix $\mathbf{D}_0$ in (9) makes the exo-Euro method entirely unfit for a more useful rectangular SUT update.[17] And, the exo-Euro specific possible drawback is that the domestic Supply (or Make) matrix is estimated outside the exo-Euro algorithm. Thus, the Make and Use matrices are not jointly derived as "an integrated part of the up-

[17] See de Mesnard (2011) on economic implications of negatives in the inverse of $\mathbf{D}_0$.

dating process" (VROB, p. 426) which ironically is one of the main general concerns of the advocates of the Euro method itself (see also point 4 in the Euro advantages list).

## 3 Should the Euro method be used in the absence of industry output?

VROB made an effort to evaluate the SUTs updating performance of the Euro and SUT-RAS variants. One of their main conclusions is that "whenever industry output is available, the SUT-RAS method *should* be used and otherwise the SUT-EURO *should* be used instead" (p. 423, emphasis added). It must be noted that using the expression "should be used" in both cases of this recommendation is unfounded, as objectively:

- the country coverage of the evaluation in question is small (including only Austria, Belgium, Italy and Spain),
- the covered time period spans from 2000 to 2005 is also short despite the availability of more recent SUT data,
- changes in inventories are not updated separately, but are taken as part of gross capital formation (which is important for checking the robustness of the approach),
- the 60-industry and 60-product classification is also a reason for concern, including not evaluating the update procedures of a more useful rectangular SUT system with more products than industries (for which all the existing Euro variants are unfit), and
- generally one should be very careful in giving "should"-type recommendations on the issue because of e.g. not covering other well-performing existing methods (which was the case in VROB assessments).

In any case an interesting question is whether the Euro method really outperforms the GRAS (SUT-RAS) method in the absence of industry output. To answer this question, I first assess the relevant SUT-Euro and SUT-RAS methods considered in VROB using the Spanish SUTs already employed in the previous sections, along with the standard SUT-RAS approach that assumes the availability of industry outputs.

Recently, Temursho et al. (2019) showed that SUT-RAS, which was introduced by TT to jointly estimate SUTs in the absence of commodity outputs, is a particular case of the GRAS approach. That is, with the appropriate formulation of the GRAS benchmark matrix $\mathbf{X}_0$ and the corresponding row and column sums constraints of $\mathbf{v}_{GRAS}$ and $\mathbf{u}_{GRAS}$, the outcome of GRAS is exactly equivalent to that of SUT-RAS. For example, for the case of SUTs at basic prices, this is true if one defines the GRAS variables as follows (with subscripts on null matrix/vectors indicating their dimension):

$$\mathbf{X}_0 = \begin{bmatrix} -\mathbf{S}_0^d & \mathbf{0}_p & \mathbf{U}_0^d & \mathbf{Y}_0^d \\ \mathbf{O}_{p\times s} & -\mathbf{m}_0 & \mathbf{U}_0^m & \mathbf{Y}_0^m \\ \mathbf{0}_s' & 0 & \mathbf{n}_{u0}' & \mathbf{n}_{y0}' \end{bmatrix}, \ \mathbf{u}_{GRAS} = \begin{bmatrix} \mathbf{0}_{2p} \\ n \end{bmatrix} \ \text{and} \ \mathbf{v}_{GRAS} = \begin{bmatrix} -\mathbf{x} \\ -m \\ \mathbf{x}-\mathbf{v} \\ \mathbf{y} \end{bmatrix}. \qquad (13)$$

Given such equivalence for any SUT-RAS variant, in what follows for convenience I will mostly use the term GRAS for SUT-RAS, though using the latter term is also correct as it spells out that GRAS is applied to a SUT framework.[18]

"The SUT-RAS method with an explicit treatment of TLS and output by industry derived endogenously" considered in VROB can be alternatively derived using GRAS with the following reformulation of the underlying required information in (13):

$$\mathbf{X}_0 = \begin{bmatrix} -\mathbf{S}_0^d & \mathbf{o}_p & \mathbf{U}_0^d & \mathbf{Y}_0^d \\ \mathbf{O}_{p\times s} & -\mathbf{m}_0 & \mathbf{U}_0^m & \mathbf{Y}_0^m \\ \mathbf{o}_s' & 0 & \mathbf{n}_{u0}' & \mathbf{n}_{y0}' \\ \hat{\mathbf{x}}_0 & \mathbf{o}_s & -\hat{\mathbf{x}}_0 & \mathbf{O}_{s\times f} \end{bmatrix}, \quad \mathbf{u}_{GRAS} = \begin{bmatrix} \mathbf{o}_{2p} \\ n \\ \mathbf{o}_s \end{bmatrix} \text{ and } \mathbf{v}_{GRAS} = \begin{bmatrix} \mathbf{o}_s \\ -m \\ -\mathbf{v} \\ \mathbf{y} \end{bmatrix}. \qquad (14)$$

This variant of GRAS without exogenous industry output, which for convenience is referred to as GRAS-0, by construction is expected to perform worse than the standard SUT-RAS or the Euro variants because *on the supply side no valid external restriction is imposed*. That is, the first $s$ columns of $\mathbf{X}_0$ in (14) are simply constrained to sum to a null vector to ensure that $\mathbf{x}' = \boldsymbol{\imath}'\mathbf{S}^d$, while the last $s$ rows of $\mathbf{X}_0$ in (14) are also constrained to sum to zero to ensure that industry gross outputs from the supply side are consistent with those derived from the demand side. Without some sort of external restriction, the base-year Supply structure cannot be fully utilized in a GRAS-0 update. However, this is not the case in the Euro approach because it uses the base-year market share matrix $\mathbf{D}_0$ in the IO quantity model (1.8) in deriving the estimate of $\mathbf{x}$. (Note, however, that both approaches make full use of the base-year Use structure.) As such it is not at all surprising that the Euro variants were found by VROB to outperform this simple GRAS-0 approach.

I replicate VROB results which are presented in Table 3, using the weighted absolute percentage error (WAPE) indicator as a measure of goodness of fit of the estimated SUT components vis-à-vis their actual counterparts.[19] As in VROB, it is found that the Euro-G generally outperforms its Euro-A variant. The latter variant's *average* WAPE (i.e. the average of corresponding WAPEs of the 8 considered projections) of the estimates of the whole intermediate Use table $\mathbf{U}$, final Use table $\mathbf{Y}$ and the domestic Supply table $\mathbf{S}^d = \mathbf{V}'$, relative to the corresponding average WAPEs of the Euro-G variant, were found to be larger by 1.2%, 0.7% and 1.7%, respectively. This confirms the general superiority of RAS-type adjustments over the "arithmetic-averaging" update, since as shown in the previous

[18]The practical advantage of the SUT-RAS method as presented in TT is or might be that the main components of SUTs are used individually and there is no need to put all of them within one integrated framework such as $\mathbf{X}_0$ and its row and column-sums as in (13), including changing the signs of certain components of SUTs.

[19]I use only WAPE because: (a) WAPE is often equivalent (in terms of rankings of matrix goodness-of-fit statistics) to the information-based psi statistic $\hat{\psi}$ widely used in geographical literature and to the simpler standardized absolute error (SAE) indicator (see Voas and Williamson, 2001, and TWY), (b) only one statistic is sufficient for the purposes of this paper, and (c) space consideration.

**Table 3:** Performance of the GRAS and Euro variants

| | 2000-01 | 2000-02 | 2000-03 | 2000-04 | 2000-05 | 2000-06 | 2000-07 | 2010-15 | *Average* |
|---|---|---|---|---|---|---|---|---|---|
| | GRAS with available industry output (Absolute WAPEs, %) | | | | | | | | |
| $\mathbf{U}$ | 9.76 | 20.08 | 21.34 | 22.64 | 25.58 | 27.54 | 27.59 | 25.17 | *22.46* |
| $\mathbf{U}^d$ | 8.56 | 15.72 | 16.79 | 18.36 | 21.63 | 23.39 | 23.64 | 24.19 | *19.04* |
| $\mathbf{U}^m$ | 14.12 | 38.56 | 40.66 | 39.42 | 41.43 | 42.53 | 41.37 | 28.60 | *35.83* |
| $\mathbf{n}_u$ | 12.59 | 20.75 | 26.74 | 28.25 | 33.51 | 39.92 | 37.26 | 27.40 | *28.30* |
| $\mathbf{Y}$ | 5.05 | 7.55 | 8.09 | 8.92 | 9.27 | 10.06 | 10.89 | 13.31 | *9.14* |
| $\mathbf{Y}^d$ | 4.41 | 6.44 | 6.66 | 7.10 | 7.97 | 8.56 | 9.52 | 12.18 | *7.85* |
| $\mathbf{Y}^m$ | 14.93 | 23.98 | 25.22 | 31.21 | 22.00 | 22.29 | 26.81 | 29.61 | *24.51* |
| $\mathbf{n}_y$ | 0.88 | 2.73 | 6.92 | 7.49 | 10.21 | 14.54 | 9.02 | 8.82 | *7.58* |
| $\mathbf{S}^d$ | 1.74 | 2.17 | 2.34 | 2.87 | 3.04 | 3.46 | 3.68 | 1.40 | *2.59* |
| $\mathbf{m}$ | 7.43 | 10.80 | 10.67 | 9.20 | 9.47 | 13.27 | 13.88 | 12.98 | *10.96* |
| | Euro-A performance (Euro-A to GRAS Relative WAPEs, %) | | | | | | | | |
| $\mathbf{U}$ | 15.8 | 15.7 | 20.2 | 22.8 | 18.5 | 20.9 | 20.3 | 17.7 | *19.0* |
| $\mathbf{U}^d$ | 16.7 | 22.1 | 26.4 | 28.5 | 25.2 | 27.0 | 24.0 | 16.3 | *23.3* |
| $\mathbf{U}^m$ | 14.2 | 5.4 | 8.8 | 14.1 | 2.2 | 5.2 | 10.3 | 23.3 | *10.4* |
| $\mathbf{n}_u$ | 10.8 | 4.6 | 20.6 | -0.6 | 29.0 | 41.7 | 41.0 | 8.0 | *19.4* |
| $\mathbf{Y}$ | 12.6 | 10.9 | 16.0 | 13.4 | 14.6 | 14.2 | 16.3 | 7.0 | *13.1* |
| $\mathbf{Y}^d$ | 15.8 | 11.6 | 18.1 | 17.2 | 14.4 | 12.9 | 17.5 | 10.6 | *14.8* |
| $\mathbf{Y}^m$ | 3.9 | 10.6 | 12.5 | 1.6 | 17.6 | 17.0 | 6.6 | -10.6 | *7.4* |
| $\mathbf{n}_y$ | -23.6 | -9.7 | 2.9 | 20.8 | 8.9 | 19.3 | 36.4 | 11.0 | *8.2* |
| $\mathbf{S}^d$ | 85.7 | 153.2 | 184.1 | 151.6 | 179.4 | 173.3 | 153.7 | 525.0 | *200.7* |
| $\mathbf{m}$ | 13.2 | 12.7 | 38.7 | 63.2 | 14.8 | 16.0 | 22.4 | 26.8 | *25.6* |
| | Euro-G performance (Euro-G to GRAS Relative WAPEs, %) | | | | | | | | |
| $\mathbf{U}$ | 15.2 | 15.2 | 19.7 | 21.4 | 15.4 | 19.4 | 18.9 | 16.1 | *17.7* |
| $\mathbf{U}^d$ | 15.9 | 21.3 | 25.7 | 27.1 | 22.0 | 25.5 | 22.3 | 14.8 | *21.8* |
| $\mathbf{U}^m$ | 14.2 | 5.3 | 8.8 | 13.0 | 1.8 | 4.8 | 10.2 | 21.5 | *9.9* |
| $\mathbf{n}_u$ | 9.5 | 2.5 | 18.6 | -3.5 | 5.6 | 32.9 | 34.0 | 6.9 | *13.3* |
| $\mathbf{Y}$ | 12.3 | 10.4 | 15.3 | 13.2 | 10.7 | 13.8 | 16.5 | 6.7 | *12.4* |
| $\mathbf{Y}^d$ | 15.5 | 11.0 | 17.5 | 17.6 | 7.9 | 11.7 | 16.9 | 10.1 | *13.5* |
| $\mathbf{Y}^m$ | 3.7 | 10.5 | 12.1 | 0.7 | 18.2 | 22.5 | 10.8 | -10.8 | *8.5* |
| $\mathbf{n}_y$ | -23.4 | -10.2 | 0.4 | 16.8 | 17.7 | 14.1 | 33.1 | 12.7 | *7.7* |
| $\mathbf{S}^d$ | 84.5 | 151.6 | 182.0 | 150.4 | 163.4 | 173.1 | 150.9 | 507.6 | *195.4* |
| $\mathbf{m}$ | 13.1 | 11.7 | 36.5 | 61.1 | 21.2 | 19.8 | 27.1 | 21.9 | *26.6* |
| | GRAS without industry outputs (GRAS-0 to GRAS Relative WAPEs, %) | | | | | | | | |
| $\mathbf{U}$ | 58.2 | 54.1 | 74.8 | 90.3 | 100.6 | 108.4 | 120.8 | 24.4 | *79.0* |
| $\mathbf{U}^d$ | 78.6 | 87.4 | 118.9 | 138.1 | 145.4 | 157.1 | 172.4 | 29.0 | *115.8* |
| $\mathbf{U}^m$ | 12.4 | -1.3 | -3.3 | 3.1 | 6.8 | 8.0 | 13.9 | 11.5 | *6.4* |
| $\mathbf{n}_u$ | 33.6 | 22.2 | 48.1 | 47.0 | 43.3 | 55.8 | 84.6 | 11.2 | *43.2* |
| $\mathbf{Y}$ | 30.3 | 43.4 | 65.7 | 84.8 | 97.8 | 113.1 | 124.7 | 12.8 | *71.6* |
| $\mathbf{Y}^d$ | 35.3 | 49.4 | 81.3 | 105.8 | 109.9 | 117.4 | 123.7 | 17.6 | *80.0* |
| $\mathbf{Y}^m$ | 6.5 | 20.0 | 26.4 | 35.6 | 73.0 | 140.7 | 127.0 | -1.8 | *53.4* |
| $\mathbf{n}_y$ | 209.9 | 101.0 | 33.7 | 56.3 | 41.8 | 28.8 | 130.7 | -21.9 | *72.5* |
| $\mathbf{S}^d$ | 266.3 | 464.6 | 593.2 | 604.5 | 711.9 | 734.6 | 740.1 | 439.8 | *569.4* |
| $\mathbf{m}$ | 10.9 | 16.3 | 21.2 | 36.7 | 83.6 | 72.9 | 85.4 | 11.1 | *42.3* |

*Note*: 2010-15 means updating a SUT component for 2015 on the base of the corresponding matrix for 2010. Method $M$ to GRAS Relative WAPE refers to $Relative\ WAPE = (WAPE_M/WAPE_{GRAS} - 1) \times 100$.

section Euro-G is a restricted variant of RAS.

Note from Table 3 that compared to GRAS with available industry output, the average WAPEs of Euro-G in estimating $\mathbf{U}$, $\mathbf{Y}$ and $\mathbf{V}$ are larger than the corresponding GRAS measures, respectively, by 17.7%, 12.4% and 195.4%. (The corresponding Euro-A to GRAS relative WAPEs are 19.0%, 13.1% and 200.7%). The particularly high relative WAPEs of the Make matrix estimates imply that the assumption of a constant market share matrix

$\mathbf{D}_0$ used in the Euro method is largely inconsistent with data and is best avoided in the practice of SUTs updating.

The GRAS-0 variant, as expected, on average performs much worse than the Euro variants, which is consistent with VROB results. The unacceptably high average relative WAPE of 569.4% in estimating $\mathbf{V}$ confirms our expectation on its general poor performance as a consequence of the absence of any supply-side restriction. Note again that in this respect the Euro method uses the base-year market share matrix that imposes certain structural constraints on the domestic Supply update. The general poor performance of GRAS-0 as presented in Table 3 implies that for all practical purposes this simple GRAS variant is useless for joint SUTs updating.

Is it thus indeed the case that there is no better alternative for Euro variants in the absence of industry output? One straightforward option out of different possibilities is to have an estimate of gross output that is then used within the standard GRAS as applied to SUTs at basic prices (or SUT-RAS) setting as in (13). Of course, the better the estimate of $\mathbf{x}$, the closer the results will be to those of GRAS with available industry output. Depending on data availability different options are possible. For example, if there is a time series of gross output by industry, one can use simple or more sophisticated forecasting methods such as exponential smoothing, time series linear models, moving average or autoregressive integrated moving average models, among many alternatives.

I consider, however, the simplest approach feasible with the data already at hand. One can estimate gross output vector from:

$$\tilde{\mathbf{x}} = \hat{\mathbf{x}}_0 \hat{\mathbf{v}}_0^{-1} \mathbf{v}, \tag{15}$$

i.e. using the gross output-to-GVA ratios of the base year, which if multiplied by the corresponding GVA forecast readily gives one estimate of $\mathbf{x}$. Besides being a much simpler approach, (15) seems to be based on a more realistic assumption than using constant market shares in $\mathbf{D}_0$ and the fixed product sales structure IO model (1.8) in deriving gross outputs within the Euro algorithm. In what follows, the GRAS variant which uses (15) as an estimate of $\mathbf{x}$ in implementing (13) is referred to as GRAS-1.

The results of GRAS-1 as compared to those of the Euro variants are presented in Table 4. Comparing GRAS-1 to Euro-G (as a superior variant of the Euro method) reveals that the GRAS-1 approach generally performs better on average across all the considered eight projections. Euro-G estimates of $\mathbf{U}$, $\mathbf{Y}$, $\mathbf{S}^d$ and $\mathbf{m}$ have larger average WAPEs of, respectively, 2.4%, 0.5%, 5.2% and 8.9% than those of GRAS-1. This implies that the claimed Euro-specific advantage of using an IO quantity model (e.g. point 8 of the Euro advantages list) can very well be a disadvantage of the method. Although there are cases when the Euro-G has better estimates of some SUT components,[20] there is enough evidence

[20] This is especially the case for the 2010-15 projection where most components of SUTs are estimated better under Euro-G than GRAS-1. The Euro method, however, is often used for updating missing SUTs

**Table 4:** Performance of GRAS-1 vis-à-vis the Euro variants

| | 2000-01 | 2000-02 | 2000-03 | 2000-04 | 2000-05 | 2000-06 | 2000-07 | 2010-15 | *Average* |
|---|---|---|---|---|---|---|---|---|---|
| | GRAS-1 performance (Euro-G to GRAS-1 Relative WAPEs, %) | | | | | | | | |
| $\mathbf{U}$ | -4.6 | 0.8 | 3.1 | 5.6 | 3.7 | 5.7 | 4.4 | 0.1 | *2.4* |
| $\mathbf{U}^d$ | -7.5 | -0.7 | 1.2 | 3.7 | 4.2 | 6.7 | 4.7 | 3.3 | *2.0* |
| $\mathbf{U}^m$ | 4.5 | 4.0 | 6.8 | 10.2 | 1.1 | -0.7 | 0.4 | -7.9 | *2.3* |
| $\mathbf{n}_u$ | -13.9 | -0.1 | 11.0 | 3.1 | 14.7 | 47.7 | 40.9 | -4.8 | *12.3* |
| $\mathbf{Y}$ | 1.2 | 1.8 | 4.5 | 0.6 | 1.7 | 1.1 | -2.7 | -4.2 | *0.5* |
| $\mathbf{Y}^d$ | 0.7 | 0.3 | 3.4 | 0.9 | -1.7 | 0.8 | -2.2 | -3.3 | *-0.1* |
| $\mathbf{Y}^m$ | 3.5 | 5.4 | 6.6 | -2.4 | 11.5 | -1.6 | -8.2 | -8.9 | *0.7* |
| $\mathbf{n}_y$ | -8.2 | 17.2 | 13.4 | 9.7 | 11.9 | 9.0 | 12.6 | -5.7 | *7.5* |
| $\mathbf{S}^d$ | -12.4 | -3.3 | 3.5 | 6.9 | 6.6 | 8.2 | -1.3 | 33.6 | *5.2* |
| $\mathbf{m}$ | 7.1 | 4.7 | 17.7 | 43.4 | 12.2 | -0.5 | -1.0 | -12.3 | *8.9* |
| | GRAS-1 with $\tilde{\mathbf{x}}_{Euro-A}$ (Euro-A to GRAS-1 Relative WAPEs, %) | | | | | | | | |
| $\mathbf{U}$ | 0.4 | 1.1 | 1.8 | 2.0 | 1.1 | 1.9 | 2.4 | 1.0 | *1.5* |
| $\mathbf{U}^d$ | -0.3 | -0.4 | -0.6 | -0.9 | -0.1 | 0.1 | 0.2 | 0.0 | *-0.2* |
| $\mathbf{U}^m$ | 2.6 | 3.9 | 5.9 | 9.1 | 1.0 | 2.2 | 3.7 | 3.9 | *4.0* |
| $\mathbf{n}_u$ | -1.5 | 6.1 | 15.4 | 1.6 | 33.2 | 46.3 | 50.2 | 4.4 | *19.5* |
| $\mathbf{Y}$ | 2.7 | 4.8 | 5.8 | 2.9 | 2.5 | 1.7 | 0.6 | -2.5 | *2.3* |
| $\mathbf{Y}^d$ | 2.6 | 4.4 | 4.8 | 2.7 | -0.1 | -1.8 | -1.8 | -2.2 | *1.1* |
| $\mathbf{Y}^m$ | 4.0 | 2.5 | 3.8 | -2.9 | 5.4 | 2.3 | -0.7 | -6.9 | *0.9* |
| $\mathbf{n}_y$ | -18.2 | 81.6 | 37.5 | 39.2 | 27.5 | 35.2 | 56.2 | 10.3 | *33.7* |
| $\mathbf{S}^d$ | -1.1 | -1.4 | 1.0 | -0.2 | -1.8 | -2.4 | -2.0 | 45.3 | *4.7* |
| $\mathbf{m}$ | -0.3 | -2.5 | 10.0 | 24.0 | -0.6 | -0.3 | 1.8 | 2.3 | *4.3* |
| | GRAS-1 with $\tilde{\mathbf{x}}_{Euro-G}$ (Euro-G to GRAS-1 Relative WAPEs, %) | | | | | | | | |
| $\mathbf{U}$ | 0.3 | 1.2 | 1.8 | 1.7 | 0.7 | 1.4 | 2.1 | 1.2 | *1.3* |
| $\mathbf{U}^d$ | -0.6 | -0.3 | -0.7 | -1.2 | -0.4 | -0.5 | -0.2 | 0.5 | *-0.4* |
| $\mathbf{U}^m$ | 2.6 | 4.0 | 6.2 | 8.8 | 2.5 | 2.4 | 3.9 | 3.0 | *4.2* |
| $\mathbf{n}_u$ | -1.8 | 5.5 | 14.6 | 0.6 | 13.3 | 39.9 | 45.2 | 3.8 | *15.1* |
| $\mathbf{Y}$ | 2.3 | 4.4 | 5.3 | 2.7 | 1.9 | 1.6 | 0.7 | -2.4 | *2.1* |
| $\mathbf{Y}^d$ | 2.0 | 4.2 | 4.6 | 3.4 | -1.5 | -1.9 | -1.6 | -2.1 | *0.9* |
| $\mathbf{Y}^m$ | 3.8 | 2.3 | 3.2 | -4.3 | 6.1 | 4.6 | 0.2 | -7.3 | *1.1* |
| $\mathbf{n}_y$ | -13.5 | 70.7 | 32.1 | 31.5 | 30.8 | 27.3 | 46.9 | 12.0 | *29.7* |
| $\mathbf{S}^d$ | -1.2 | -1.5 | 0.6 | 0.1 | -1.5 | -2.0 | -2.2 | 49.2 | *5.2* |
| $\mathbf{m}$ | -0.3 | -3.1 | 9.1 | 24.9 | 6.7 | 3.0 | 4.1 | -0.4 | *5.5* |

*Note*: See notes to Table 3.

*against* the claim that in the absence of industry outputs "the SUT-Euro should be used".

However, one can actually go further in comparing the two approaches. What if in-

in-between two available benchmark tables, for which purposes GRAS-1 seems to be largely superior as follows from the results in Table 4. When there are two benchmark SUTs available and there is a missing SUT in-between, there are *other simple options* of obtaining the estimate of $\mathbf{x}$ in its absence that use the relevant benchmark information. Assume one needs to project SUTs for $k$ years between the start and ending periods of available SUTs. Then for year $t \in (0, k+1)$ with missing SUTs take

$$\tilde{\mathbf{x}}_t = \left(\frac{k+1-t}{k+1}\hat{\mathbf{x}}_{start}\hat{\mathbf{v}}_{start}^{-1} + \frac{t}{k+1}\hat{\mathbf{x}}_{end}\hat{\mathbf{v}}_{end}^{-1}\right)\mathbf{v}_t \quad \text{or} \quad \tilde{\mathbf{x}}_t = \frac{k+1-t}{k+1}\mathbf{x}_{start} + \frac{t}{k+1}\mathbf{x}_{end},$$

where subscript *start* and *end* refer to the two available benchmark SUTs, considered as starting and ending period of SUTs interpolation. Such a time-weighted estimate roughly accounts for the trend of, respectively, industry output-to-GVA ratio and industry output movements under a "normal development" scenario assumption. In fact, one could apply the same time-weighting procedure to the GRAS benchmark matrix $\mathbf{X}_0$ as well in order to, at least partially, account for possible structural change that affects the structure and denseness of SUT components. (The idea of time-weighting for SUT update purposes goes back at least to TWY, see footnote 8, p. 94).

stead of (15), in GRAS-1 the gross outputs as endogenously obtained by the Euro-A and Euro-G methods were used? That is, how well do the two perform, if a practitioner by some chance ends up with exactly the same estimates of $\mathbf{x}$ as those generated by Euro-A and Euro-G variants, denoted respectively by $\tilde{\mathbf{x}}_{Euro-A}$ and $\tilde{\mathbf{x}}_{Euro-G}$? The results, presented in the middle and bottom parts of Table 4, again indicate that GRAS-1 generally outperforms both Euro variants. These tests have important consequences for the Euro method: its general under-performance in such imagined (hypothetical) circumstances reveals the problematic nature of the method's underlying assumptions and/or its nature of adjustments of a SUT's individual elements. Three points need to be briefly mentioned in this respect:

- The largest errors show up in Euro's estimation of net taxes on products both for intermediate and final uses. This confirms the ad-hoc nature of equal treatment of both positive and negative entries in the Euro update procedure, as already discussed in Section 2.1.
- The estimates of imported intermediate Use and total imports show the second largest (consistent) errors, which further substantiate the criticism raised with respect to the restrictive nature of the imports (row) multipliers and intermediate Use (column) multipliers of the standard Euro method.
- Even though gross outputs are exactly the same, Euro's estimates of $\mathbf{V}$ show considerably much larger error than GRAS-1 for the 2010-15 projection, clearly pointing to the inadequacy of using a constant market shares assumption.

All in all, here the conclusion is that there are other methods - at least certain variants of GRAS - that perform better than the Euro method even under the circumstances of missing industry output data.

## 4 On the issue of (un)fair comparisons

One of the crucial motivations of the VROB study is the belief of the authors that in the existing evaluation studies, specifically TT, "the SUT-EURO and the SUT-RAS methods, as defined by these authors, do not actually use the same information, thus leading to *unfair comparisons and misleading conclusions*" (VROB, p. 425, emphasis added). Strangely enough the study by TWY is not explicitly criticized in this respect (though cited), although it covers more updating methods (i.e. eight different methods, some of which have more than one variant) and also finds the Euro method among the worst performing SUT updating approaches.

I appreciate the work done by VROB and agree that in general the performance of methods in the presence of "asymmetry in the information used by the different methods ... cannot be attributed only to the methods themselves but also to the different information used as the starting point" (p. 425). However, I cannot agree with such statements as regards the Euro method, which is VROB's main concern, due to the following *facts* and their implications:

1. From its introduction in 2002, the Euro method was explicitly stated to be superior to the traditional RAS technique. Specifically mentioned were its claimed features of avoiding arbitrary adjustments of input coefficients and of rational updating of individual coefficients that is in line with the trend of technology and market forces. See also the full Euro advantages list in the Introduction.[21] It thus might have been expected that comparative assessments of the Euro method would have appeared in the intervening years from, at least, interested IO research community to verify whether such claims really hold true in practice. In fact, it was surprising to claim Euro superiority over RAS without providing any convincing justification, including theoretical and/or empirical support.
2. The most important statistical office of the European Union - Eurostat - has been using (extensively) the Euro variants for at least 17 years now since its first publication came out (see e.g. Beutel, 2002, 2008; Eurostat, 2008, 2011; United Nations, 2018). However, since Eurostat's "mission is to provide *high quality statistics* for Europe" (excerpt from the official webpage of Eurostat, emphasis added), it must be of Eurostat's interest to use robust statistical methodologies, including those used for updating/estimating missing SUTs of Member States. From this perspective, the Euro method does not stand on the same footing with other more or less data-demanding updating techniques. Moreover, the "limited data requirements" was understood as one of the Euro advantages. Crucially, avoiding or allowing for ad hoc adjustments in updated SUTs might well have important consequences for real-world policy assessments and recommendations that are based on such data analysis.
3. One of the first tasks before constructing the by now well-known World Input-Output Database (WIOD, Timmer, 2012) was to carry out an extensive comparative assessment of the existing (and most promising) IOT/SUT updating techniques. Since the Euro method was used by Eurostat and essentially advocated in Eurostat (2008), there was little doubt on its inclusion in the evaluation exercises (and as such it was in the running to be the method chosen to be used in the project for updating the missing national SUTs of the WIOD countries, which was the main aim of these evaluations). The studies by TWY and TT are the final results of this particular task.
4. The data requirements of all methods evaluated are explicitly spelled out in both TWY and TT, thus leaving little room for misinterpretation of the results. For example, TT wrote that "The Euro method requires that the use table at basic prices is distinguished between domestic and imported intermediate and final uses. Otherwise all the approaches require almost the same data availability for the projection year tab-

[21]All these advantages are also mentioned in Beutel (2002).

les except for the Euro method that does not use outputs by industry and computes them endogenously" (pp. 875-876). Then immediately in the footnote it is stated that: "This is, in fact, one of the drawbacks of the Euro method, since it does not use total outputs by industry that are also available from the national accounts. We believe it is important to use this extra information rather than estimating it." Consequently, there was no base for essentially accusing TT of unfair comparisons and misleading conclusions. These statements have actually indirectly suggested ways to improve the Euro method. (This is probably one of the reasons that the exo-Euro variant was introduced, but it is prone to similar criticisms as discussed briefly in Section 2.2 above.)

5. Finally, the conclusions of TWY and TT proved *not* to be misleading even under a "fair comparison" environment as understood by VROB since Section 3 of this paper shows that a simple GRAS variant generally outperformed the Euro method even in the absence of industry outputs. There is no doubt that there are other GRAS variants that have this property, but for the purposes of this paper one empirical counterexample is enough to make the point.

From a more subjective perspective, there are *prominent* IO and regional economists who often critically assessed the Euro method on its theoretical background – even well before the studies of TWY and TT – in formal IO-related meetings, workshops and conferences, and for justification of their claims openly have been calling for extensive comparative empirical assessments of the method. Just one example, Prof. Jan Oosterhaven (who kindly agreed to my request of mentioning his name in this respect) had and still has this concern, and as far as I remember (going back to, at least, the period of my PhD studies in 2005-2009) was always very critical of the method, primarily on theoretical grounds.

## 5 Concluding remarks

This paper presents a closer look into the nature of adjustments of SUT elements as implemented by the Euro method variants. The analytical and empirical assessment of the method's adjustments and underlying assumptions shows that the Euro method is largely an ad-hoc method. Some of the critical issues discussed are the following:

- Product imports (row) multipliers are defined to be proportional to GVA growth rates, which was not supported empirically in case of Spain,
- The realistic possibility of having negative multipliers with unfavorable consequences,
- The possibility of obtaining zero or undefined multipliers requiring additional ad-hoc treatment,
- The restrictive nature of adjustments of SUT elements in both variants of the Euro update procedure,
- Equal treatment of positive and negative elements that may easily lead to large struc-

tural deviations of the updated SUT compared to its benchmark,

- Domestic intra-industry intermediate transactions are updated (multiplied) simply by the corresponding GVA ratios in both Euro variants without industry outputs, which was not empirically supported in case of Spain,
- The Euro method is not applicable for the update of a far more useful rectangular SUT system with a larger number of products than industries,
- The reliance of the Euro update on assumed constant market shares (or commodity output proportions) is at odds with empirical data, while it is also highly likely that the use of a specific IO quantity model is a disadvantage rather than advantage of the method.

Clearly, some of the criticisms covered in this paper could be addressed in order to improve the method. For example, one could start adjusting negative elements by the inverse of the corresponding multipliers – whether cell-specific or of bi-proportional nature – in line with the GRAS updating technique, or use growth rates of imports in defining the imports (row) multipliers. Doing so, however, makes the use of better-performing standard GRAS (SUT-RAS) more appealing, while the superior Euro variants based on a geometric mean option would become a GRAS variant themselves. The result of the latter scenario is that the Euro method would essentially lose its "Euro" identity and become a particular but still restricted case of a more flexible, theoretically sound and better performing GRAS approach.

In view of all the issues raised in this paper, the hope is that our colleagues at Eurostat might decide to simply switch to better-performing GRAS variant(s) with weaker (equivalently, more realistic) underlying assumptions in updating the missing SUTs of the Member States. In case of unavailable industry outputs, it seems that one of the simplest and most reasonable approaches is first to estimate industry outputs exogenously, which by construction allows for more control over the estimation of such critical data (compared to a method that endogenously derives industry outputs within a non-robust update procedure that is based on questionable assumptions). Then simply use this estimate together with other usual exogenous data to update/estimate the missing SUTs within the standard GRAS framework. One such simple approach is shown to generally outperform the Euro method in this paper. Two other options (not empirically assessed) for estimating industry outputs in case of missing SUTs in-between two available benchmark tables are also suggested. The missing industry outputs could be alternatively estimated using other, including more sophisticated, methods.

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